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Entropy generation in a plane turbulent oscillating jet

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Abstract

The entropy generation in a plane turbulent jet is revisited. This flow is characterized by quasi-periodic lateral oscillations, documented in the literature, due to the instability of the flow. Based on the laws of Thermodynamics, an analysis of the entropy generation has been presented by Bejan [Entropy Generation Minimization. The Method of Thermodynamic Optimization of Finite-Size Systems and Finite-Time Processes, Wiley, New York, 1996, p. 61]. In this paper, a term has been added that takes into account the experimentally observed oscillations. The results are compared for the cases with oscillations and without oscillations.

Keywords: Plane turbulent jets; Entropy generation; Flow oscillations

1. Introduction

Turbulent flows are characterized by fluctuations that are generally considered as random and are therefore described by means of their statistical properties. This random nature of turbulent flows is apparent for the small scales of the flow, but it may be not for the large scales, as it has been suggested in the literature during the last two or three decades. This fact is especially remarkable in the case of free turbulent shear flows – jets, wakes and mixing layers – where it has been detected a coherent, and to a certain extent, deterministic behavior [1,2]. In many cases, oscillations with clearly distinctive frequencies have been detected and measured [3,4].

No matter what the origin of the oscillations could be, the evolution of the flow is always accompanied by thermodynamic entropy generation. This generation intuitively increases with the entrainment at the flow boundaries, as a result of the mixing and lateral extension of the flow with the surrounding, quiescent fluid. Bejan [5] has recently developed the theoretical basis to quantify the entropy generation in a variety of physical situations of interest in engineering, as it is the case of a plane turbulent jet.

In this paper, a modified version of the model developed by Bejan for a plane turbulent jet is presented. It is included now the energy associated to the observed lateral oscillations of the flow.

2. Lateral oscillations of plane turbulent jets

There are many instances in Fluid Mechanics, where repetitive phenomena, almost periodic, are present with clearly distinctive frequencies [6]. This is the case of the lateral oscillations in a plane turbulent jet, whose characteristic frequencies, when scaled with the appropriate local variables, compose a unique dimensionless parameter.

The origin of these oscillations is the instability of the flow, either at the first stages of the viscous regime, or rather, once the turbulent regime has been established, when the flow penetrates and drags the surrounding fluid, experiencing a kind of buckling or lateral oscillation.

In the first case – the laminar flow, where the oscillations are easily detectable and quantifiable by means of various experimental techniques – the conventional analysis of the problem is the one derived from the theory of the linear stability of the flow. The existence of a defined disturbance is postulated, superimposed to a known laminar flow, and the behavior of the disturbance, in time and space, is analyzed. If it grows the flow

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Nomenclature

e	oscillations energy
D	diameter
f	local frequency of oscillations
v	longitudinal flow velocity
u	transversal flow velocity
u_c	centerline flow velocity
T_c	centerline flow temperature
$f(\zeta)$	similarity shape profiles
$\bar{x}$	dimensionless coordinate
$\bar{u}_c$	dimensionless velocity
$\bar{D}$	dimensionless diameter
$\Delta\bar{T}_c$	dimensionless temperature difference
h	enthalpy
S_{gen}	entropy generation rate
$\bar{S}_{gen}$	dimensionless entropy generation rate
$\bar{S}_{gen,max}$	maximum dimensionless entropy generation rate
St	Strouhal number

Greek symbol

α	empirical constant for entrainment

is unstable; if the disturbance decays, then the flow is considered stable.

On the other hand, the observation and experimental characterization of the oscillations in turbulent shear flows require the use of specialized instrumentation and visualization techniques [1,7], in addition to the processing of random signals [3]. The results of numerous investigations during the last 30 years seem to confirm that the large scales of turbulent shear flows are to a certain extent, deterministic [2,9].

In this way, it has been detected an almost periodic behavior of turbulent shear flows as coherent structures and vortex pairing in mixing layers and two-dimensional wakes, and as flapping and oscillations in plane and circular jets. In all the cases a frequency of oscillation associated to a characteristic wavelength can be quantified, that scales with the flow width and the average local velocity of the fluid, giving rise to a representative constant Strouhal number for the whole flow [4,8].

A complete theory about the presence of these periodic structures is still lacking, but it is true that they could be considered as if they were harmonic oscillators with defined energy characteristics [4]. Thus, their oscillation energy can be written as [10]

$$e = \frac{1}{2}(2\pi D f)^2, \tag{1}$$

where D represents the cross-section scale of the flow (width of the jet or the wake) and f is the local frequency of the lateral oscillations.

3. Entropy generation in a plane turbulent jet

The evolution of any shear flow always goes accompanied by entropy generation, which intuitively increases as a function of entrainment at the flow boundaries, as well as of mixing and lateral extension of the flow with the surrounding, still fluid. Bejan [5] has recently developed the theoretical basis to quantify the entropy generation in a variety of physical situations of interest like a turbulent plane jet. Based on the laws of Thermodynamics for a control volume of thickness dx, through which a turbulent plane jet flows, Fig. 1, it is

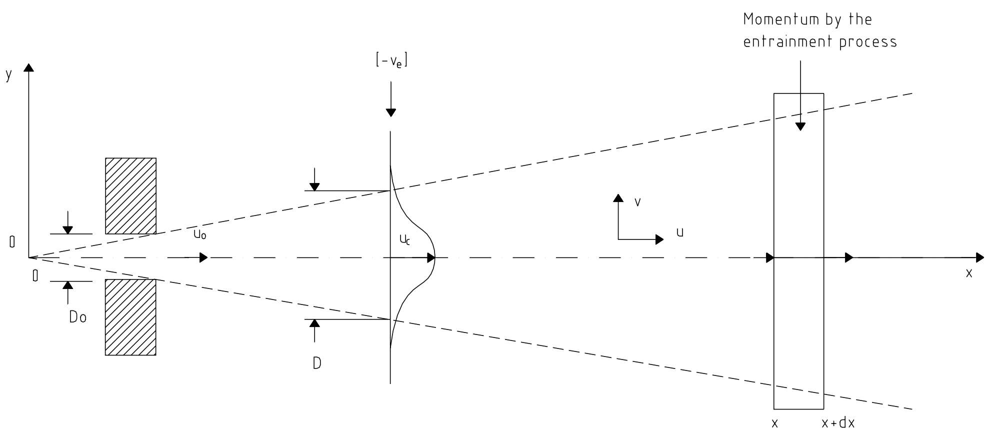

Fig. 1. Schematic view of a plane turbulent jet.

possible to establish a relationship between the similarity characteristics of the flow and the entropy generation that accompanies the shearing process.

As it is well-known [11], turbulent shear flows are characterized by *similarity* laws in their velocity, temperature and other local properties profiles. A shape function, $f(\zeta)$, can thus be proposed for the profile of the local average velocities

$$u = u_c f(\zeta), \tag{2}$$

where $\zeta = y/D$ is the cross-section coordinate and $u_c(x)$ is the velocity at the center of the jet. By simplicity, it is possible to assume that the profile of average temperatures has the same form that one of the velocities

$$T - T_\infty = (T_c - T_\infty) f(\zeta), \tag{3}$$

where $T_c(x)$ is the temperature at the centerline of the jet. In the summarized analysis that follows, some dimensionless variables are used:

$$\bar{x} = \frac{x}{D_0}, \quad \bar{u}_c = \frac{u_c}{u_0}, \quad \bar{D} = \frac{D}{D_0}, \quad St = \frac{fD}{u_c}, \tag{4}$$

where St is the Strouhal number that characterizes the lateral oscillations of the jet. Based on the work by Morton et al. [12], Bejan [5] integrated the boundary layer type equations of continuity and momentum in the x-direction, for a turbulent plane jet, under the similarity consideration given by Eq. (2), obtaining

$$\bar{u}_c = \left(\frac{I_1}{4\alpha I_2}\right)^{1/2} \bar{x}^{-1/2}, \tag{5}$$

$$\bar{D} = \frac{4\alpha}{I_1}\bar{x}, \tag{6}$$

where the entrainment hypothesis in a shear flow has been used. The entrainment process, that is, the advance of the turbulence interface into the outer fluid, influenced by lateral mean convection [13], essentially means that the transverse velocity component can be expressed as $-v_\infty \approx \alpha u_c$, being α an empirical constant for any self-preserving flow, but different from one kind to another. For a plane jet, it has a value of around 0.04 [11], and Bejan [pp. 63 et seq.] concluded that a proper empirical value to be used in his integral analysis is 0.0367, based on mixing-length modeling of the jet. In fact, through a pure first law of Thermodynamics analysis, he obtained an optimal value of α in the same order of magnitude as the empirical entrainment coefficient generally reported in the open literature.

In Eqs. (5) and (6), and in the expressions that appear ahead in this paper, certain coefficients arise which represent integrals of the similarity profile shape assumed for the velocities and temperatures

$$I_1 = \int_{-\infty}^{\infty} f \,\mathrm{d}\zeta, \quad I_2 = \int_{-\infty}^{\infty} f^2 \,\mathrm{d}\zeta, \quad I_3 = \int_{-\infty}^{\infty} f^3 \,\mathrm{d}\zeta. \tag{7}$$

Thus, according to the first law of Thermodynamics, it is possible to write

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}x}\int_{-\infty}^{\infty}\left[h + \frac{u^2}{2}\right]\rho u \,\mathrm{d}y - \left[h_\infty + \frac{(-v_0)^2}{2}\right]\frac{\mathrm{d}}{\mathrm{d}x} \\ &\times \int_{-\infty}^{\infty}\rho u\,\mathrm{d}y + \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}x}\int_{-\infty}^{\infty}[2\pi D f]^2 \rho u\,\mathrm{d}y \\ &= 0.\end{aligned} \tag{8}$$

The first term expresses the net increase in the flow direction, in the control volume, of energy (represented by the local averages of enthalpy and kinetic energy). The second term considers the energy contribution to the jet by the process of lateral extension into the ambient fluid. These two terms were originally derived by Bejan [5]. The third term – proposed and deducted in this investigation – represents the energy associated to the oscillations observed in the flow, in agreement with Eq. (1).

Eq. (8) is integrated with respect to x, assuming that the flow is isothermal as it comes out from the nozzle. In this way, an expression for the dimensionless temperature difference in the central line of the jet is obtained,

$$\Delta\bar{T}_c = \frac{I_3 + \alpha^2 I_2 + (2\pi St)^2 I_1}{I_2^2}\left[\left(\frac{1}{4\alpha\bar{x}}\right)^{1/2} - \frac{1}{4\alpha\bar{x}}\right], \tag{9}$$

where $\Delta\bar{T}_c$ is defined as

$$\Delta\bar{T}_c = \frac{c_p}{u_0^2/2}(T_c - T_\infty). \tag{10}$$

On the other hand, from the second law of Thermodynamics for the volume of control defined in Fig. 1, the entropy generation along the jet can be written as

$$\frac{\mathrm{d}S_{gen}}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}x}\int_{-\infty}^{\infty}\rho u s\,\mathrm{d}y - s_\infty\frac{\mathrm{d}}{\mathrm{d}x}\int_{-\infty}^{\infty}\rho u\,\mathrm{d}y \geqslant 0 \tag{11}$$

and can be reduced to

$$\frac{\mathrm{d}S_{gen}}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}x}\int_{-\infty}^{\infty}\frac{c_p}{T_\infty}(T - T_\infty)\rho u\,\mathrm{d}y. \tag{12}$$

Considering again the similarity profiles for the velocity and temperature, Eqs. (2) and (3), respectively, and the integral given by Eq. (7), the generation of entropy can be expressed as

$$\frac{\mathrm{d}\bar{S}_{gen}}{\mathrm{d}\bar{x}} = \frac{\mathrm{d}}{\mathrm{d}\bar{x}}(I_2\bar{u}_c\bar{D}\Delta\bar{T}_c), \tag{13}$$

where

$$\bar{S}_{gen} = \frac{S_{gen}}{\rho u_0^3 D_0/2T_\infty}. \tag{14}$$

Integrating Eq. (13) from the nozzle outlet (where $\Delta\bar{T}_c = 0$), it is obtained

$$\bar{S}_{gen} = I_2\bar{u}_c\bar{D}\Delta\bar{T}_c. \tag{15}$$

Finally, replacing $\Delta \bar{T}_c$ from the first law analysis, Eq. (9), it is obtained

$$\bar{S}_{gen} = \frac{I_3 + \alpha^2 I_2 + (2\pi St)^2 I_1}{I_2^{3/2}} \left[1 - \left(\frac{I_1}{4\alpha \bar{x}} \right)^{1/2} \right]. \tag{16}$$

4. Results

Eq. (16) expresses the entropy generation rate, $\bar{S}_{gen}$, for a plane turbulent jet as a function of the longitudinal coordinate $\bar{x}$. It includes as parameters, the entrainment constant of the flow with the surrounding fluid at rest, α, the integrals previously defined with the velocity and temperature similarity profiles, Eq. (7), and most important, the energy associated to the observed lateral oscillations of the flow, through the Strouhal number. Without considering the latter effect, this equation reduces to the one presented by Bejan [5], for the non-oscillating jet.

The equation shows that the entropy generation rate grows as $\bar{x}$ increases. In order to calculate this tendency it is necessary to use appropriate similarity profiles for the velocity and the temperature in Eqs. (2), (3) and (7). Table 1 summarizes the calculations performed in the present work for two typical forms of the similarity profiles of a plane jet reported in the literature: one corresponding to a square hyperbolic tangent function (used by Bejan in his analysis), and the other corresponding to a square hyperbolic secant function. The table also shows the Strouhal numbers that take into account the experimentally observed frequencies in these flows, according to Cervantes and Goldschmidt [3]. Included in this table are the calculated values of the integrals given by Eq. (7).

The longitudinal evolution of the entropy generation in the jet is shown in Fig. 2. The results without oscillations (lower curves of Fig. 2) agree well with the trends reported by Bejan [5] (Fig. 3.13, p. 68, not shown in the present paper, for clarity of figure), for various simple models of the velocity profile: triangular, parabolic, top hat and Gaussian profile shapes.

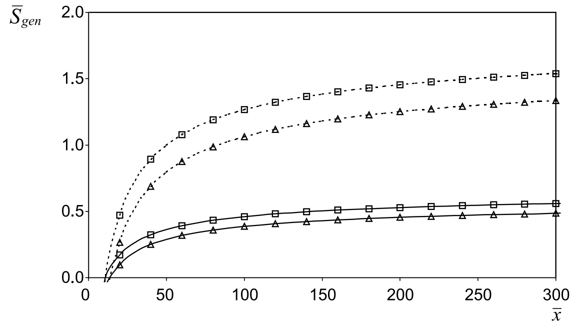

Fig. 2. Entropy generation for a plane jet: (- - -) with oscillations, (—) without oscillations; (△) profile of Bejan [5], (□) profile of Townsend [11].

On the other hand, the results for the oscillating jet (upper curves in Fig. 2) correspond to the physically expected behavior. As can be established from the *Constructal Theory* formulated by Bejan [14], turbulence represents the natural tendency of a flow field to seek and find a flow structure that enhances the mixing rate. The participation of the oscillations in the flow gives as a result that part of the flow energy must contribute to the entrainment and mixing process of turbulence. This means that higher values for the entropy generation are to be obtained in each jet cross-section due to the lateral oscillations, as compared to the calculated entropy generation for the case without oscillations. Moreover, in both cases there is a sudden increase in the entropy generation, in the flow region where $\bar{x}$ is about 20, that is, where the jet is more unsteady and has the more distorted profiles, right before it reaches the self-preserving region. This response is enhanced by the flow oscillations.

In the similarity region of the jet ($\bar{x} > 20$), a smooth increase of the entropy generation with respect to the longitudinal coordinate must be observed, as in Fig. 2. Self-preservation asserts that a moving equilibrium is set-up in which the conditions at the initiation of the flow are largely irrelevant, and the flow depends on few simple parameters and is geometrically similar at all sections, Townsend [11]. According to this renowned

Table 1
Effects of the Strouhal number in the entropy generation of a turbulent plane jet

Similarity profile	Strouhal number[a]	I_1	I_2	I_3	$\bar{S}_{gen,max}$ Eq. (17)	
					Without oscillations	With oscillations
$f(\zeta) = 1\tanh^2(\zeta)$[b]	0.154[a]	2[b]	4/3[b]	16/15[b]	0.694[b]	1.91[c]
$f(\zeta) = \mathrm{sech}^2(\zeta(2/\pi)^{1/2})$[d]	0.154[a]	2π[c]	1.67[c]	1.33[c]	0.62[c]	1.706[c]

[a] Cervantes and Goldschmidt [3].
[b] Bejan [5].
[c] Present work.
[d] Townsend [11].

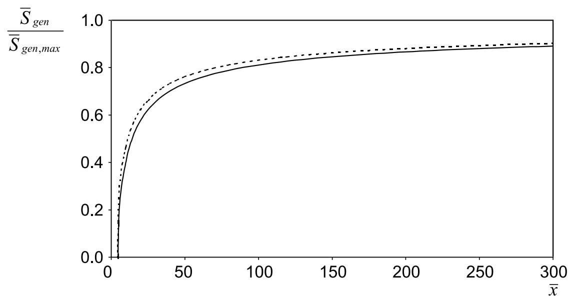

Fig. 3. Normalized entropy generation with oscillations, according to Eq. (17) and Table 1: (- - -) profile of Bejan [5], (—) profile of Townsend [11].

author in the turbulent shear flows theory, self-preservation may exist only as an asymptotic condition. This seems to be the case with respect to entropy generation as seen in Fig. 3, where a similarity behavior can be observed after normalizing the ordinates of Fig. 2, with the maximum value of the entropy generation (last column of Table 1) obtained for large values of $\bar{x}$ in Eq. (16), that is,

$$\bar{S}_{\mathrm{gen,max}} = \frac{I_3 + \alpha^2 I_2 + (2\pi St)^2 I_1}{I_2^{3/2}}. \tag{17}$$

5. Concluding remarks

The production of entropy in a plane turbulent jet, taking into account its natural oscillations, has been presented. The analysis consisted of a modified version of the model developed by Bejan [5] for a plane jet, including this time, the energy associated to the experimentally observed lateral oscillations of the flow. The calculated results are congruent and confirm what intuitively can be expected: entropy generation grows along the flow direction and depends directly on the entrainment with the still ambient fluid. This response is enhanced with the natural oscillations of the jet. A strong increment in entropy generation could be detected in the highly unsteady region of the flow (that is, where $\bar{x} \approx 0.2$). On the other hand, the smooth increase of the production of entropy in the downstream direction, towards an asymptotic maximum value, confirms the similarity properties of the flow in the far field region. This self-preserving behavior of the oscillating turbulent plane jet is difficult to characterize with respect to the entropy generation and needs further work. Nevertheless, it has not been previously reported in the literature.

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